New Specification

Rewarding Learning

ADVANCED
General Certificate of Education

Further Mathematics

Assessment Unit A2 2

assessing

Applied Mathematics

[AFM21]

Assessment

MARK SCHEME

General Marking Instructions

GCE Advanced/Advanced Subsidiary Further Mathematics

Introduction

The mark scheme normally provides the most popular solution to each question. Other solutions given by candidates are evaluated and credit given as appropriate; these alternative methods are not usually illustrated in the published mark scheme.

The marks awarded for each question are shown in the right-hand column and they are prefixed by the letters M, W and MW as appropriate. The key to the mark scheme is given below:

M indicates marks for correct method.

W indicates marks for working.

MW indicates marks for combined method and working.

The solution to a question gains marks for correct method and marks for accurate working based on this method. Where the method is not correct no marks can be given.

A later part of a question may require a candidate to use an answer obtained from an earlier part of the same question. A candidate who gets the wrong answer to the earlier part and goes on to the later part is naturally unaware that the wrong data is being used and is actually undertaking the solution of a parallel problem from the point at which the error occurred. If such a candidate continues to apply correct method, then the candidate's individual working must be followed through from the error. If no further errors are made, then the candidate is penalised only for the initial error. Solutions containing two or more working or transcription errors are treated in the same way. This process is usually referred to as "follow-through marking" and allows a candidate to gain credit for that part of a solution which follows a working or transcription error.

Positive marking

It is our intention to reward candidates for any demonstration of relevant knowledge, skills or understanding. For this reason we adopt a policy of following through their answers, that is, having penalised a candidate for an error, we mark the succeeding parts of the question using the candidate's value or answers and award marks accordingly.

Some common examples of this occur in the following cases:

(a) a numerical error in one entry in a table of values might lead to several answers being incorrect, but these might not be essentially separate errors;

(b) readings taken from a candidate's inaccurate graphs may not agree with the answers expected but might be consistent with the graphs drawn.

When the candidate misreads a question in such a way as to make the question easier only a proportion of the marks will be available (based on the professional judgement of the examining team).

Section A

1. (i) MW1 Correct amplitude

   (ii) M1 Tries to use correct formula
   W1 Correct value of ω found
   MW1 Correct value of period found

   (iii) M1 Tries to use correct formula
   W1 Correctly substituted
   W1 Correct speed found

   (iv) M1 Tries to use correct formula
   W1 Correctly substituted (allow ft for their values of r, w)
   W1 Correct time found – cao

2. (i) M1 Tries to find masses
   W1 3 correct masses
   M1 Tries to find CG
   W1 3 correct values
   M1 Tries to multiply mass by distance
   W1 3 correct values
   M1 Tries to equate moments
   W2 Each side of equation correct
   W1 Correct value of $\bar{y}$

   (ii) M1 Trying to use tan to calculate angle
   W2 Correct numerator and denominator – see below
   W1 Correct angle – cao

   Notes
   [A] Allow ft for their value of $\bar{y}$, provided used correctly

3. (i) MW1 Suitable explanation for $R_B = 0$

   (ii) M1 Tries to take moments about G
   W1 Each side of equation correct
   W1 Correct Friction
   M1 Tries to resolve horizontally
   W2 Each side of equation correct
   M1 Tries to resolve vertically
   W2 Each side of equation correct
   M1 Tries to divide
   MW1 Correctly substitutes Fr in terms of R_A
   M1 Tries to simplify equation
   W1 Correct value of v

4. (i) M1 Tries to set up $F = ma$
W1 Includes $-T$ and $-kv$ correctly
M1 Tries to use Hooke's law for tension
W1 Correct value of tension
W1 Correct value of dashpot force
MW1 Rearranges equation correctly
W1 Correct final answer

(ii) M1 Knows that underdamping indicates no real roots
M1 Tries to set up $b^2 - 4ac < 0$
W1 Correct inequality
W1 Correct value of λ

(iii) M1 Tries to substitute 0.9 into AQE
W1 Correct equation
W2 2 correct values for m
M1 Tries to set up correct equation for x
W1 Correct equation

5. (i) MW1 Finds BF = 2m
MW1 Uses ABG and FBG are congruent/ABF is isosceles to give AB = 2m

(ii) M1 Tries to use moments
W1 Finds $R_D = 30$
M1 Tries to resolve vertically (or take moments)
W1 Correct value of R_A

(iii) M1 At E, tries to resolve perp to DF
W1 Finds $T_{CE} = 0$
M1 At C, tries to resolve perp to BD
W1 Resolves correctly
W1 Finds $T_{FC} = 0$

(iv) M1 Tries to resolve vertically at D (or use triangle of forces)
W1 Correct value of T_{DE}
MW1 Identifies as thrust
M1 Tries to resolve horizontally at D (or use triangle of forces)
W1 Correct value of T_{CD}
MW1 Identifies as tension
M1 Tries to resolve horizontally at C (or use triangle of forces)
W1 Correct value of T_{BC}
MW1 Identifies as tension

Section B

1. (i) MW1 Finds correct angle CAB
M1 Tries to resolve horizontally
W1 Correct answer
MW1 Obtains correct vertical component
M1 Tries to find resultant
W1 Correct value
MW1 Finds correct angle

	(ii)	M1	Tries to equate moments about A (or B)
		W1	Correct equation (allow ft for their R from (i))
		W1	Correct conclusion
2.	(i)	M1	Tries to differentiate
		W3	Each correct term
	(ii)	M1	Tries to integrate
		W3	Each correct term
		MW1	Uses initial conditions to find correct **c**
		MW1	Uses $t = 4$ to get correct value of **s**
		M1	Tries to use Pythagoras
		W1	Correct distance
	(iii)	M1	Sets **i** component equal to 0
		W1	Correct solution
		MW1	Finds correct solution to **k** component equation
		MW1	Selects correct answer
3.	(i)	M1	Tries to use conservation of momentum
		W1	Correct equation
		M1	Tries to use Newton's law of restitution
		W1	Correct equation
		M1	Tries to solve equations
		W1	First correct velocity
		MW1	Second correct velocity
	(ii)	MW1	Uses conservation of momentum (allow ft of their $\mathbf{v}_B$ from (i))
		MW1	Uses Newton's law of restitution (allow ft of their $\mathbf{v}_B$ from (i))
		MW1	First correct velocity – cao
		MW1	Second correct velocity – cao
	(iii)	M1	Tries to compare velocities
		W1	Correct solution to inequation
		W1	Correct conclusion
4.		M1	Tries to set up $F = ma$
		W2	Each side of equation correct
		MW1	Rearranges equation into a suitable format
		M1	Tries to find IF
		W1	Correct IF
		MW1	Multiplies through by IF
		M1	Tries to integrate
		W2	Each side of equation correct
		M1	Tries to use $t = 0$, $v = 9$
		W1	Correct constant
		M1	Tries to find v when $t = 30$
		W1	Correct speed
5.	(i)	M1	Tries to integrate πxy^2
		W1	Correct limits
		MW1	Correct substitution for y^2
		MW2	Each of 2 terms integrated correctly

W1 Correct answer to substitution of limits
M1 Tries to equate moments
W1 Correct answer

(ii) M1 Tries to find masses
W1 Correct values
M1 Tries to find distances
W1 Correct values
M1 Tries to find mass by distance
W1 Correct values
M1 Tries to equate moments
W1 Correct equation
W1 Correct answer

(iii) M1 Knows that COM lies vertically above A
M1 Use of tan
W1 Correct values substituted (allow ft of their $\overline{y}$ from (i))
W1 Correct angle – cao

Section C

1. (i) M1 Attempts to find E(X)
W1 Correct constants used
W1 Correct answer
M1 Attempts to find Var(X)
MW1 Correct constants used
W1 Correct answer

(ii) MW1 Recognises normal distribution
MW1 Correct mean and variance

(iii) M1 Knowing normal
MW1 Correct variance
M1 Using $P(\overline{X} > 8.5)$
MW1 Correct z – value (or evidence of use of tables/calculator)
M1 Using 1 – ….
W1 Correct answer

2. M1 Correct null hypothesis
M1 Correct alternative hypothesis
MW1 Correct column and row totals
M1 Trying to find an expected frequency
W1 Correct
MW1 Correct 2nd expected frequency
W1 2 more expected frequencies correct
W1 All remaining expected frequencies correct
M1 Trying to use test statistic formula
W1 2 correct values in table
W1 Remaining values correct
W1 Correct χ^2 value
MW1 Correct v value
MW1 Correct critical value
MW1 Statement comparing test and critical values
MW1 Correct conclusion

3. (i) M1 Attempts to find E(D)
W1 Correct value
M1 Attempts to find Var(D)
W1 Correct 1st constant
W1 Correct 2nd constant
W1 Correct value
MW1 Correct z – value (or suitable evidence of use of tables/calculator)
M1 Using P($D > 0$)
MW1 Using 1 – …
W1 Correct answer

(ii) MW1 Comment about assumed value of variance
MW1 Suggestion of why this may not be a reasonable assumption

4. (i) MW1 Correct mean of A
M1 Attempts to find $S_A{}^2$
W1 Correct answer
MW1 Correct mean of B
MW1 Correct $S_B{}^2$
M1 Attempts to use pooled variance estimator formula
W1 Correct substitutions (allow ft of their $S_A{}^2$ and $S_B{}^2$)
W1 Correct answer – cao

(ii) M1 Correct null hypothesis
M1 Correct alternative hypothesis
M1 Trying to use t-test
MW1 Correct value for v
MW1 Correct critical value
M1 Trying to find test statistic
MW1 Correct substitutions (allow ft of their values from (i))
W1 Correct value $t_{\text{test}} = -1.942$
MW1 Comparing test and critical values
MW1 Correct conclusion

5. (i) M1 Trying to find mean
W1 Correct value of mean
M1 Trying to find S^2
W1 Correct value of S^2
M1 Trying to use formula for CI
MW1 Use of 1.96
W1 Correct substitutions (allow ft of their x and S^2)
W1 Lower limit = 22.1
W1 Upper limit = 22.9

(ii) M1 Trying to use correct formula for width
W1 Correct substitutions
W1 Correct value of z
MW1 Correct value for P($Z < 1.656$)
M1 Using 1 – …
W1 Correct %

Section D

1. (a) M1 Uses correct principle
W1 Correct answer

(b) (i) M1 Knows how to find number of edges
W1 Correct value
W1 Subtracts to get correct answer

(ii) MW1 8 colourings correct
MW1 All correct

(c) MW1 Pairings of each set used
MW1 Correct

2. (i) MW1 All correct

(ii) MW1 EF times correct
MW1 ES times correct
MW1 LS times correct
MW1 LF times correct

(iii) MW1 Correct answer

(iv) MW1 Correct expected time
M1 Using correct formula
W1 Correct value
M1 Trying to use normal distribution
W1 Correct z – value (or evidence of method if using calculator)
M1 Trying to find $P(Z > \cdots)$
W1 Correct answer

3. (i) MW1 Correct values

(ii) M1 Trying to differentiate
W1 Correct answer

(iii) MW1 Correct values

(iv) M1 Trying to substitute $t = 1/5$
W1 Correct answer

(v) MW1 Correct function differentiated
M1 Correct choice of value for t
MW1 Correct multiple of derivative
W1 Correct answer

4. (i) M1 Correct method used for counting ways of placing
W1 Correct answer

(ii) M1 Knowing to use answer to (i)
M1 Trying to square answer to (i)
W1 6 correct terms of expansion
W1 All correct terms of expansion
W1 Simplified correctly

(ii) M1 Attempting an availability matrix
W1 Correct answer
MW1 Identifies polynomial of excluded positions
M1 Uses Rook IE
M1 Attempts to use formula
W1 Correct substitutions
W1 Correct answer

5. (a) (i) M1 Attempts to form index
W1 Correct answer

(ii) M1 Correct value for x_i selected
MW1 Correct substitution
W1 Correct answer

(b) (i) M1 Correct index chosen
M1 Correct replacement chosen
W1 Correct substitutions

(ii) M1 Correct choice of terms
MW1 1st term correct
MW1 Choice of 2nd term
MW1 2nd term correct
W1 Correct answer

6. (i) M1 Uses slack variables
MW1 Inequalities in correct form
MW1 Objective function in correct form

(ii) MW1 1st row correct
MW1 2nd row correct
MW1 3rd row correct

(iii) M1 Attempts to find pivot
MW1 Correctly
MW1 Divides pivot row correctly
MW1 Reduces other rows correctly
MW1 Identifies pivot row correctly
MW1 Divides pivot row correctly
MW1 Reduces other rows correctly

(iv) M1 Reads results from final column
W1 x, y correct
W1 P correct

Section A Mechanics 1

			AVAILABLE MARKS
1 **(i)**	Amplitude = 2 m	MW1	
(ii)	$a = r\omega^2$		
	$\Rightarrow 6 = 2\omega^2$	M1	
	$\Rightarrow \omega^2 = 3$	W1	
	$\Rightarrow T = \dfrac{2\pi}{\sqrt{3}}$	MW1	
	$= 3.6275987\ldots$		
	$= 3.63\,\text{s}$ (3 s.f.)		
(iii)	$v^2 = \omega^2 (r^2 - x^2)$		
	$\Rightarrow v^2 = 3(2^2 - 0.5^2)$	M1 W1	
	$\Rightarrow v = \dfrac{3\sqrt{5}}{2}$	W1	
	$= 3.3541\ldots$		
	$= 3.35\,\text{m}\,\text{s}^{-1}$ (3 s.f.)		
(iv)	$x = r \sin \omega t$		
	$\Rightarrow 0.5 = 2 \sin\sqrt{3}\,t$	M1 W1	
	$\Rightarrow \sqrt{3}\,t = \sin^{-1}\left(\dfrac{0.5}{2}\right)$		
	$\Rightarrow t = 0.146\,\text{s}$ (3 s.f.)	W1	10

2 **(i)**

Shape	Rectangle	Triangle	Circle	
Mass	$2400m$	$90m$	$36\pi m$	M1 W1
Distance of CG from CD	20	26	10	M1 W1
Mass × distance	$48000m$	$2340m$	$360\pi m$	M1 W1

AVAILABLE MARKS

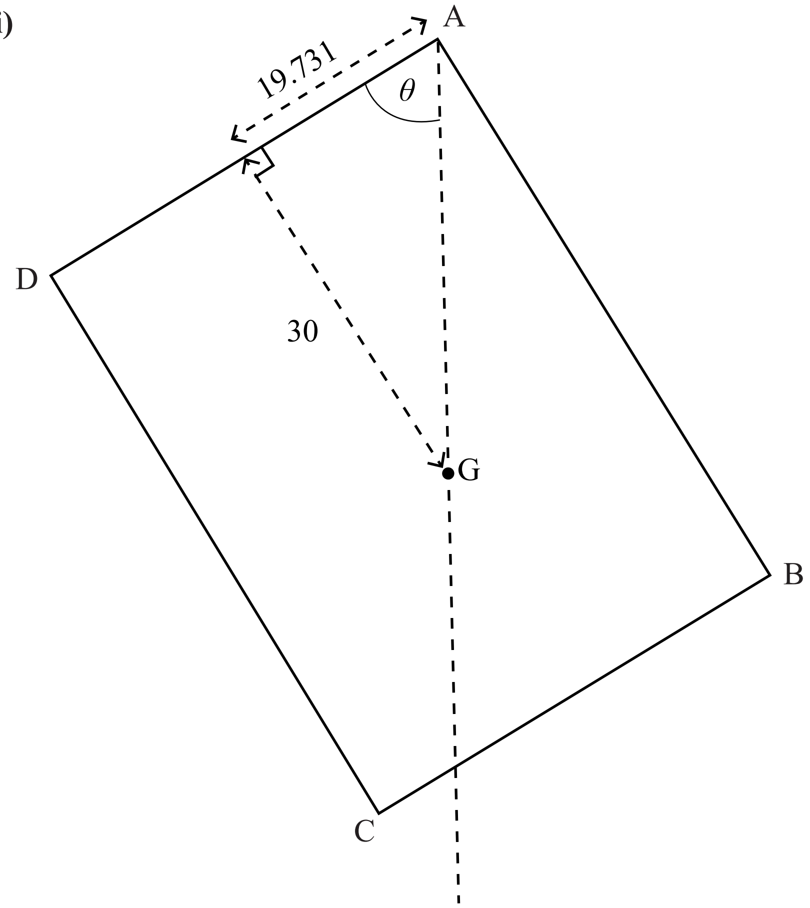
$\Rightarrow (2400m - 90m - 36\pi m)\bar{y} = (48000m - 2340m - 360\pi m)$ M1 W2

$\Rightarrow \bar{y} = \dfrac{45660 - 360\pi}{2310 - 36\pi}$

$= 20.26900\ldots$

$= 20.3\text{cm (3 s.f.)}$ W1

(ii)

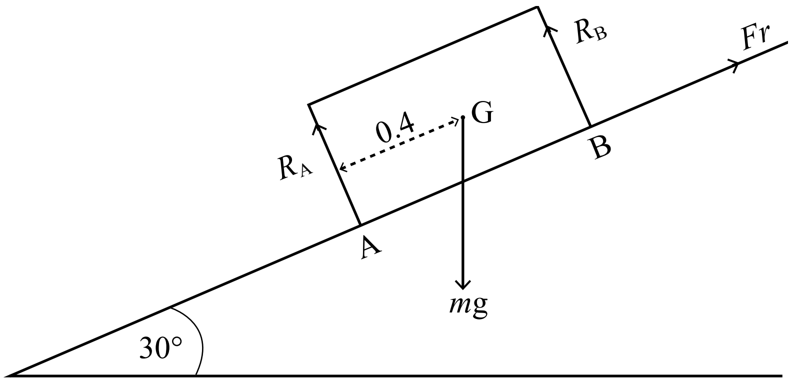


$\theta = \tan^{-1}\left(\dfrac{30}{40 - 20.269}\right)$ M1 W1 W1

$= 56.66708\ldots.$

$= 56.7° \text{ (3 s.f.)}$ W1 14

3



AVAILABLE MARKS

(i) $R_B = 0$ since the wheel is about to lose contact with the ground. MW1

(ii) M(G): $0.4 \times R_A = 1 \times Fr$ M1 W1

$\Rightarrow Fr = 0.4\,R_A$ W1

(←) $R_A \sin30° - Fr\cos30° = \dfrac{mv^2}{20}$ M1 W2

(↑) $R_A \cos30° + Fr\sin30° = mg$ M1 W2

$\Rightarrow \dfrac{R_A\sin30° - 0.4R_A\cos30°}{R_A\cos30° + 0.4R_A\sin30°} = \dfrac{mv^2}{20mg}$ M1 MW1

$\Rightarrow \dfrac{\tan30° - 0.4}{1 + 0.4\tan30°} = \dfrac{v^2}{20g}$ M1

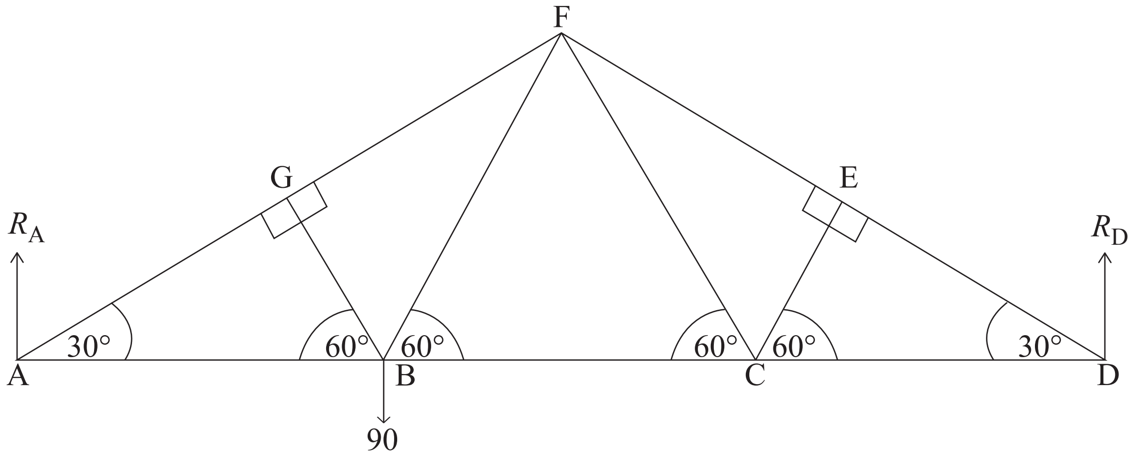
$\Rightarrow v^2 = \dfrac{20g(\tan30° - 0.4)}{1 + 0.4\tan30°}$

$\Rightarrow v = 5.31$ (3 s.f.) W1 14

			AVAILABLE MARKS
4	**(i)** $-T - kv = 0.6a$	M1 W1	
	$\Rightarrow -\frac{\lambda x}{0.25} - 3v = 0.6a$	M1 W1 W1	
	$\Rightarrow 0.6\ddot{x} + 3\dot{x} + 4\lambda x = 0$	MW1	
	Divide by 0.6		
	$\Rightarrow \ddot{x} + 5\dot{x} + \frac{20}{3}\lambda x = 0$	W1	
	(ii) $m^2 + 5m + \frac{20}{3}\lambda = 0$		
	Underdamping $\Rightarrow$ no real roots	M1	
	$\Rightarrow 25 - 4\left(\frac{20}{3}\lambda\right) < 0$	M1 W1	
	$\Rightarrow 25 < \frac{80\lambda}{3}$		
	$\Rightarrow \lambda > \frac{15}{16}$ (or $\lambda > 0.9375$)	W1	
	(iii) $\lambda = 0.9$		
	$\Rightarrow m^2 + 5m + 6 = 0$	M1 W1	
	$\Rightarrow (m + 2)(m + 3) = 0$		
	$\Rightarrow m = -2, -3$	W2	
	$\Rightarrow x = Ae^{-2t} + Be^{-3t}$	M1 W1	17

5

AVAILABLE MARKS



(i) Triangle BFC is equilateral ⇒ BF = 2m MW1
∠GBF = 60°
⇒ Triangles ABG and FBG are congruent or Triangle ABF is isosceles
⇒ AB = 2m MW1

(ii) M(A): $90 \times 2 = R_D \times 6$ M1
⇒ $R_D = 30$ N W1

(↑) $R_A + R_D = 90$ M1
⇒ $R_A = 60$ N W1

(iii) At E, resolve at right angles to FD ⇒ $T_{CE} = 0$ N M1 W1
At C, resolve at right angles to BD ⇒ $T_{FC} \cos 30° + T_{CE} \cos 30° = 0$ M1 W1
Since $T_{CE} = 0$ then $T_{FC} = 0$ N W1

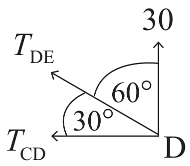
(iv) At D
(↑) $T_{DE} \cos 60° + 30 = 0$ M1
⇒ $T_{DE} = -60$N which is a thrust W1 MW1

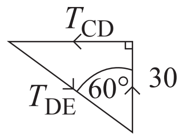
(←) $T_{CD} + T_{DE} \cos 30° = 0$ M1
⇒ $T_{CD} = 30\sqrt{3}$ N which is a tension W1 MW1

At C
(←) $T_{CD} = T_{BC}$ M1
⇒ $T_{BC} = 30\sqrt{3}$ N which is a tension W1 MW1

Alternative Solution

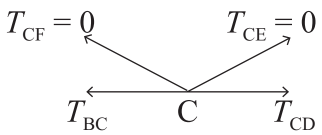
At D:





		AVAILABLE MARKS
Resultant force = 0		
Hence T_{CD} is a tension and T_{DE} is a thrust	MW2	
$T_{CD} = 30 \tan 60°$	M1	
$T_{CD} = 30\sqrt{3}$ N	W1	
$T_{DE} = \dfrac{30}{\cos 60°}$	M1	
$T_{DE} = 60$N	W1	

At C:



$T_{CD} = T_{BC}$	M1	
$\Rightarrow T_{BC} = 30\sqrt{3}$ N which is a tension	W1 MW1	20
	Section A	**75**

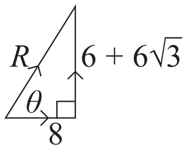
Section B Mechanics 2

AVAILABLE MARKS

1 **(i)** Angle CAB $= \tan^{-1}\left(\frac{\sqrt{3}}{1}\right) = 60°$ MW1

$(\rightarrow)$ $8 - 6 + 12\cos 60° = 8$ M1 W1

$(\uparrow)$ $6 + 12\sin 60° = 6 + 6\sqrt{3}$ MW1



$R = \sqrt{8^2 + (6 + 6\sqrt{3})^2}$ M1

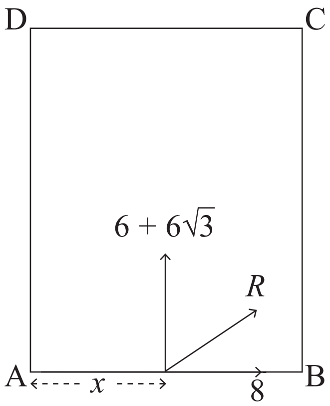
$= \sqrt{332.707\ldots}$ W1

$= 18.2$ N (3 s.f.)

$\theta = \tan^{-1}\left(\frac{6 + 6\sqrt{3}}{8}\right)$

$= 64.0°$ (3 s.f.) MW1

(ii)



M(A): $6 \times 1 + 6 \times \sqrt{3} = (6 + 6\sqrt{3})x$ M1 W1

$\Rightarrow x = 1$

i.e. R acts through B W1 10

Alternative solution

M(B): $12 \times 1 \times \sin 60° - 6 \times \sqrt{3}$ M1 W1

$= 0$

Hence R acts through B W1

			AVAILABLE MARKS
2	**(i)** $\mathbf{a} = \dfrac{d\mathbf{v}}{dt}$	M1	
	$= (-6t\mathbf{i} + 4\mathbf{j} + 3t^2\,\mathbf{k})\ \text{m s}^{-2}$	W3	
	(ii) $\mathbf{s} = \int \mathbf{v}\,dt$	M1	
	$\mathbf{s} = (-t^3 + 12t)\mathbf{i} + (2t^2 - t)\mathbf{j} + (\frac{1}{4}t^4 - 8t)\mathbf{k} + \mathbf{c}$	W3	
	$t = 0,\ \mathbf{s} = \mathbf{i} + 2\mathbf{j} \Rightarrow \mathbf{c} = \mathbf{i} + 2\mathbf{j}$	MW1	
	$t = 4 \Rightarrow \mathbf{s} = -15\mathbf{i} + 30\mathbf{j} + 32\mathbf{k}$	MW1	
	$\Rightarrow$ distance $= \sqrt{15^2 + 30^2 + 32^2}$	M1	
	$= \sqrt{2149}$		
	$= 46.4$ m (3 s.f.)	W1	
	(iii) When moving parallel to **j** the **i** and **k** components of velocity are zero and the **j** component is non-zero		
	$\Rightarrow -3t^2 + 12 = 0$ and $t^3 - 8 = 0$	M1 W1	
	$\Rightarrow t = \pm 2$ $\quad\Rightarrow t = 2$	MW1	
	When $t = 2$ the **j** component is 7 (i.e. $\neq 0$)		
	Hence the particle moves parallel to **j** when $t = 2$	MW1	16

			AVAILABLE MARKS
3	**(i)** A B		
	$m_A = 6m$ $m_B = 4m$		
	$u_A = u$ $u_B = 0$		
	$6mu = 6mv_A + 4mv_B$	M1 W1	
	$\Rightarrow 3u = 3v_A + 2v_B$		
	and		
	$-0.25u = v_A - v_B$	M1 W1	
	① + 2 × ② $\Rightarrow 2.5u = 5v_A$	M1	
	$\Rightarrow v_A = 0.5u$	W1	
	$\Rightarrow v_B = 0.75u$	MW1	
	(ii) B C		
	$m_B = 4m$ $m_C = m$		
	$u_B = 0.75u$ $u_C = 0$		
	$4m \times 0.75u = 4mv_B + mv_C$		
	$\Rightarrow 3u = 4v_B + v_C$ ③	MW1	
	$-0.75eu = v_B - v_C$ ④	MW1	
	③ + ④ $\Rightarrow 3u - 0.75eu = 5v_B$		
	$\Rightarrow v_B = 0.15u\,(4 - e)$	MW1	
	$\Rightarrow v_C = 0.6u\,(1 + e)$	MW1	
	(iii) A B		
	$0.5u$ $0.15u(4 - e)$		
	$\rightarrow$ $\rightarrow$		
	If $v_A > v_B$ then A and B will collide		
	$\Rightarrow 0.5u > 0.15u(4 - e)$	M1	
	$\Rightarrow e > \frac{2}{3}$	W1	
	$\Rightarrow \frac{2}{3} < e \leqslant 1$	W1	14

			AVAILABLE MARKS
4	$10e^{-0.02t} - 3v = 100a$	M1 W1 W1	
	$\Rightarrow 100\dfrac{dv}{dt} + 3v = 10e^{-0.02t}$		
	$\Rightarrow \dfrac{dv}{dt} + 0.03v = 0.1e^{-0.02t}$	MW1	
	$IF = e^{\int 0.03 dt} = e^{0.03t}$	M1 W1	
	$\Rightarrow e^{0.03t}\dfrac{dv}{dt} + 0.03ve^{0.03t} = 0.1e^{0.01t}$	MW1	
	Integrate		
	$\Rightarrow ve^{0.03t} = 10e^{0.01t} + c$	M1 W2	
	$\Rightarrow v = 10e^{-0.02t} + ce^{-0.03t}$		
	$t = 0,\ v = 9 \Rightarrow 9 = 10 + c$	M1	
	$\Rightarrow c = -1$	W1	
	$\Rightarrow v = 10e^{-0.02t} - e^{-0.03t}$		
	$t = 30,\ v = 10e^{-0.6} - e^{-0.9}$	M1	
	$= 5.08\ \text{m s}^{-1}$ (3 s.f.)	W1	14

			AVAILABLE MARKS
5	**(i)** $\frac{2}{3}\pi\rho r^3 d = \rho\int_0^r \pi x y^2 \mathrm{d}x$	M1 W1	
	$= \rho\pi\int_0^r x(r^2 - x^2)\,\mathrm{d}x$	MW1	
	$= \rho\pi\int_0^r (r^2 x - x^3)\,\mathrm{d}x$		
	$= \rho\pi\left[\frac{1}{2}r^2x^2 - \frac{1}{4}x^4\right]_0^r$	MW2	
	$\Rightarrow \frac{2}{3}\pi\rho r^3 d = \frac{1}{4}\rho\pi r^4$	M1 W1	
	$\Rightarrow d = \frac{3\rho\pi r^4}{2 \times 4\pi\rho r^3}$		
	$\Rightarrow d = \frac{3r}{8}$	W1	

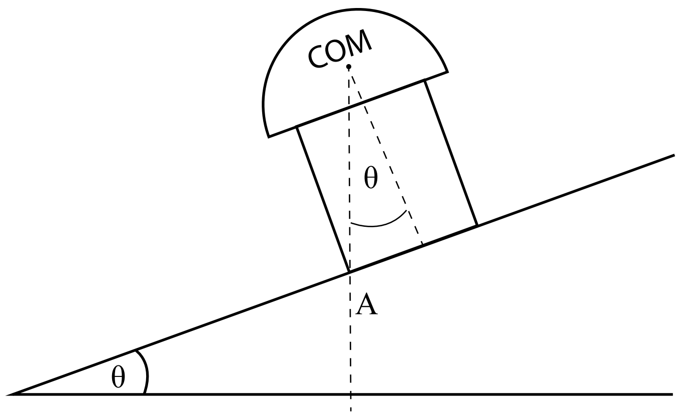
(ii)

Shape	Hemisphere	Cylinder	
Mass	$\frac{2}{3}\pi\rho(3a)^3$ $= 18\pi\rho a^3$	$\pi\rho a^2(3a)$ $= 3\pi\rho a^3$	M1 W1
Displacement of CG from plane	$\frac{3}{8}(3a) = \frac{9}{8}a$	$-\frac{3}{2}a$	M1 W1
Mass × Distance	$\frac{81}{4}\pi\rho a^4$	$-\frac{9}{2}\pi\rho a^4$	M1 W1

$\Rightarrow 21\pi\rho a^3\,\bar{y} = \frac{63}{4}\pi\rho a^4$ — M1 W1

$\Rightarrow \bar{y} = \frac{3a}{4}$ — W1

(iii)



Centre of mass lies vertically above A — M1

$\Rightarrow \tan\theta = \frac{a}{3a + \frac{3}{4}a}$

$\tan\theta = \frac{4}{15}$ — M1 W1

$\Rightarrow \theta = 14.9°$ (3 s.f.) — W1 — 21

Section B — **75**

Section C Statistics

AVAILABLE MARKS

1 **(i)** $E(X) = E(2W - 0.5Y)$

$= 2E(W) - 0.5E(Y)$ M1 W1

$= 2(5) - 0.5(4)$

$= 8$ W1

$\text{Var}(X) = \text{Var}(2W - 0.5Y)$

$= 2^2\text{Var}(W) + 0.5^2\text{Var}(Y)$ M1 MW1

$= 4(0.5) + 0.25(4)$

$= 3$ W1

(ii) For random samples taken from a (non-normal) population with finite mean μ and variance σ^2, the distribution of the sample mean $\bar{X}$ is approximately normal.

$\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$

provided that the sample size n is large. MW2

(iii) $\bar{X} \sim N\left(8, \frac{3}{40}\right)$ M1 MW1

$P(\bar{X} > 8.5) = P\left(Z > \frac{8.5 - 8}{\sqrt{\frac{3}{40}}}\right)$ M1

$= P(Z > 1.826)$ MW1

$= 1 - P(Z < 1.826)$ M1

$= 1 - \Phi(1.826)$

$= 0.0340$ (3 s.f.) W1 14

		AVAILABLE MARKS
2 H_0 : School age group and level of attendance are independent	M1	
H_1 : School age group and level of attendance are not independent	M1	

Observed frequencies

	Good	Satisfactory	Poor	Total
Junior	46	22	12	80
Senior	24	20	16	60
Total	70	42	28	140

MW1

If H_0 is true then the expected number of juniors with good attendance is

$\frac{80 \times 70}{140} = 40$ M1 W1

If H_0 is true then the expected number of juniors with satisfactory attendance is

$\frac{80 \times 42}{140} = 24$ MW1

Expected frequencies

	Good	Satisfactory	Poor	Total
Junior	40	24	16	80
Senior	30	18	12	60
Total	70	42	28	140

W2

Test statistic: $\chi^2 = \sum \frac{(O-E)^2}{E}$

O	E	$\frac{(O-E)^2}{E}$
46	40	$\frac{9}{10}$
22	24	$\frac{1}{6}$
12	16	1
24	30	$\frac{6}{5}$
20	18	$\frac{2}{9}$
16	12	$\frac{4}{3}$

M1 W2

$\chi^2 = \frac{217}{45} = 4.82$ W1

2×3 contingency table i.e. $h = 2$ and $k = 3$

$v = (h-1) \times (k-1) = 1 \times 2 = 2$ MW1

Critical value: $\chi^2_{5\%}(2) = 5.991$ MW1

$\chi^2 < \chi^2_{5\%}(2)$ MW1

i.e. there is insufficient evidence at the 5% level to indicate a dependence between age group and level of attendance. MW1 16

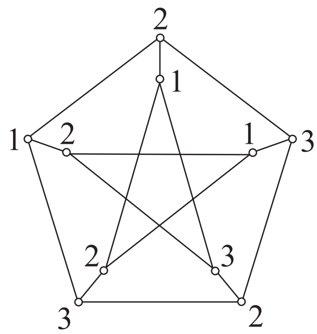
			AVAILABLE MARKS
3	**(i)**	$X \sim N(55, 4^2)$	
		$E(D) = 3.5E(X) - 3E(X)$ — M1	
		$= 0.5E(X)$	
		$= 27.5$ grams — W1	
		$\text{Var}(D) = 3.5^2\text{Var}(X) + 3\text{Var}(X)$ — M1 W2	
		$= 15.25\text{Var}(X)$	
		$= 244 \text{ grams}^2$ — W1	
		$z = \frac{0 - 27.5}{\sqrt{244}} = -1.761$ — MW1	
		$P(D > 0) = P(Z > -1.761)$ — M1	
		$= 1 - P(Z < -1.761)$ — MW1	
		$= 1 - \Phi(-1.761)$	
		$= 1 - [1 - \Phi(1.761)]$	
		$= 0.961$ (3 s.f.) — W1	
	(ii)	It has been assumed that the variance of the king size bar is greater than the variance of the standard size bar by a factor of 3.5^2 — MW1	
		This value for the variance of the king size bar is likely to be much smaller in practice. — MW1	12

			AVAILABLE MARKS
4 **(i)**	$n_A = 7 \quad \bar{x}_A = \frac{520.1}{7} = 74.3$	MW1	
	$S_A^2 = \frac{7}{6}\left(\frac{38740.85}{7} - 74.3^2\right) = 16.23\dot{6}$	M1 W1	
	$n_B = 9 \quad \bar{x}_B = \frac{520.1}{7} = 78.2$	MW1	
	$S_B^2 = \frac{9}{8}\left(\frac{55161.96}{9} - 78.2^2\right) = 15.6$	MW1	
	Pooled variance estimator is given by:		
	$S_p^2 = \frac{(n_A - 1)S_A^2 + (n_B - 1)S_B^2}{n_A + n_B - 2}$	M1	
	$S_p^2 = \frac{6(16.23\dot{6}) + 8(15.6)}{14}$	W1	
	$S_p^2 = 15.873$	W1	
(ii)	$H_0 : \mu_A = \mu_B$	M1	
	$H_1 : \mu_A < \mu_B$	M1	
	Small samples ∴ t-test (lower one-tailed)	M1	
	$v = 14$ ∴ $T \sim t(14)$	MW1	
	Critical value at 5% with $v = 14$ is $t_c = -1.761$	MW1	
	Test statistic: $T = \frac{\bar{x}_A - \bar{x}_B - (\mu_A - \mu_B)}{S_p\sqrt{\frac{1}{n_A} + \frac{1}{n_B}}}$	M1	
	If H_0 is true then $\mu_A - \mu_B = 0$, and the test value is given by		
	$t_{test} = \frac{\bar{x}_A - \bar{x}_B}{S_p\sqrt{\frac{1}{n_A} + \frac{1}{n_B}}} = \frac{74.3 - 78.2}{\sqrt{15.873}\sqrt{\frac{1}{7} + \frac{1}{9}}}$	MW1	
	$t_{test} = -1.942$	W1	
	$t_{test} < t_c$ ∴ reject H_0	MW1	
	i.e. there is sufficient evidence at the 5% level to claim that the new format enables faster texting.	MW1	18

		AVAILABLE MARKS
5 **(i)** $\bar{x} = \dfrac{900}{40} = 22.5$	M1 W1	
$S^2 = \dfrac{40}{39}\left(\dfrac{20330}{40} - 22.5^2\right) = \dfrac{80}{39}$	M1 W1	
95% confidence interval for μ given by		
$\text{CI} = \bar{x} \pm 1.96\,\dfrac{S}{\sqrt{n}}$	M1 MW1	
$\text{CI} = 22.5 \pm 1.96 \times \dfrac{\sqrt{\frac{80}{39}}}{\sqrt{40}}$	W1	
CI = (22.1, 22.9) (3 s.f.)	W2	
(ii) The width of the confidence interval is given by $\text{Width} = 2 \times z \times \dfrac{S}{\sqrt{n}}$	M1	
$0.75 = 2z \times \dfrac{\sqrt{\frac{80}{39}}}{\sqrt{40}}$	W1	
$\Rightarrow z = 1.656$	W1	
$P(Z < 1.656) = 0.9511$	MW1	
$1 - 0.9511 = 0.0489$	M1	
$100\% - (2 \times 4.89\%) = 90\%$ (nearest integer)	W1	15
	Section C	**75**

Section D Discrete and Decision Mathematics

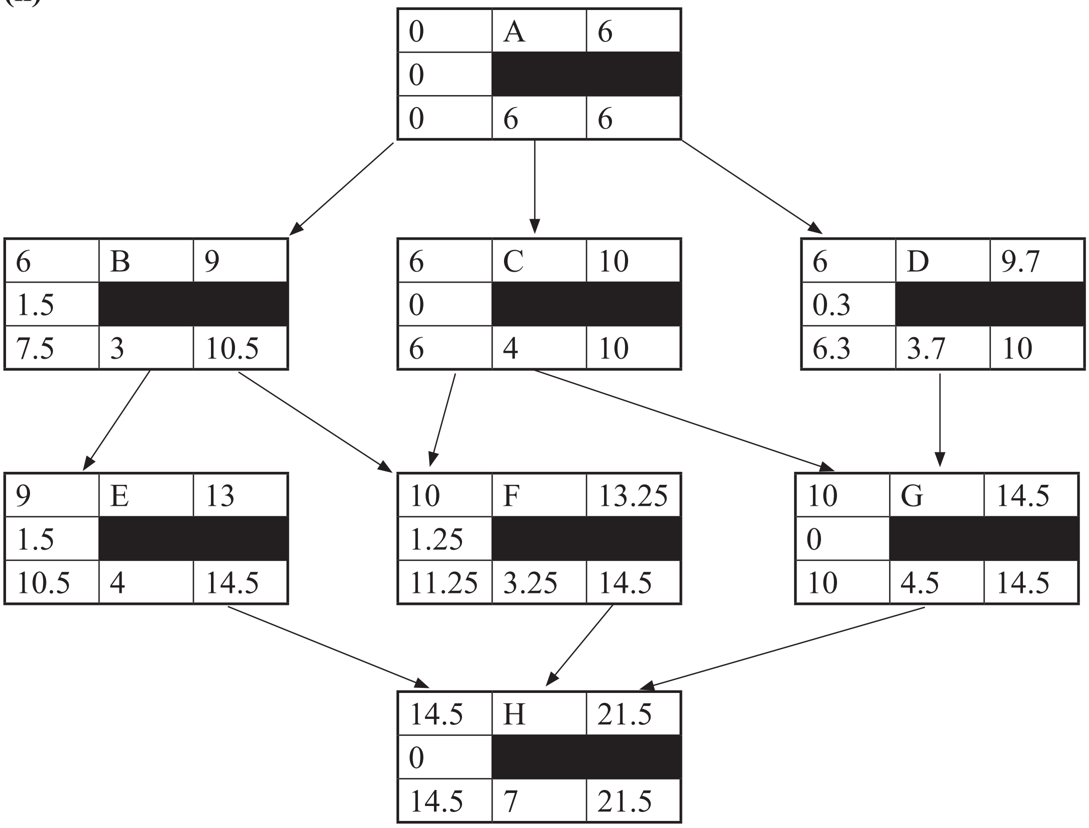
			AVAILABLE MARKS
1 **(a)**	All vertices connected, therefore all different So 6 colours	M1 W1	
(b) **(i)**	In K_{10} each vertex is joined to 9 others	M1	
	Thus K_{10} has $\frac{10 \times 9}{2} = 45$ edges	W1	
	Thus $45 - 15 = 30$ edges more	W1	
(ii)	Any acceptable colouring on the Petersen graph, e.g.,		



		MW2	
(c)	A – H B – G C – I D – J E – L F – K	MW2	9

2 **(i)** 4, 3.25, 4.5, 7 MW1

(ii)



Key:

ES	node	EF
Slack		
LS	dur	LF

ES = Early Start Time
LS = Late Start Time
EF = Early Finish Time
LF = Late Finish Time
dur = Expected time

EF times MW1
ES times MW1
LS times MW1
LF times MW1

(iii) Critical path = A C G H MW1

(iv) Expected time = 6 + 4 + 4.5 + 7 = 21.5 weeks MW1

Projected variance = $(16 + 9 + 25 + 9)/36 = 59/36 = 1.63\dot{8}$ M1 W1

$Z = (23 - 21.5)/\sqrt{1.63\dot{8}} = 1.1717$ M1 W1

$P(Z > 1.1717) = 0.121$ M1 W1 13

AVAILABLE MARKS

3 **(i)** $\frac{d}{dt}[f(t)] = a_1 + 2a_2t + 3a_3t^2 + 4a_4t^3 + ...$

$a_1, 2a_2, 3a_3, 4a_4$ MW1

(ii) $\frac{d}{dt}\left(\frac{1}{1-t}\right) = \frac{1}{(1-t)^2}$ M1

So $\frac{1}{(1-t)^2} = 1 + 2t + 3t^2 + 4t^3 + ...$ W1

(iii) 1, 2, 3, 4, ... MW1

(iv) $t = \frac{1}{5}$ M1

Sum $= 1 \div \left(\frac{4}{5}\right)^2 = \frac{25}{16}$ W1

(v) $\frac{d}{dt}\left(\frac{1}{(1-t)^2}\right) = 2 + 6t + 12t^2 + 20t^3 + ...$ MW1

$t = \frac{1}{3}$ M1

Sum $= \frac{1}{3}\frac{d}{dt}\left(\frac{1}{(1-t)^2}\right)$ MW1

$= \frac{1}{3}\frac{2}{(1-t)^3}$

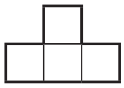
$= \frac{2}{3} \div \left(\frac{2}{3}\right)^3$

$= \frac{9}{4}$ W1 10

AVAILABLE MARKS

4 **(i)** 0 rooks 1 way
1 rook 4 ways
2 rooks 2 ways — M1
So the rook polynomial is $1 + 4x + 2x^2$ — W1

(ii) Rook polynomial for



is $1 + 4x + 2x^2$ — M1
Combined rook polynomial is
$(1 + 4x + 2x^2)^2$ — M1
$(1 + 4x + 2x^2)(1 + 4x + 2x^2)$
$= 1 + 4x + 2x^2 + 4x + 16x^2 + 8x^3 + 2x^2 + 8x^3 + 4x^4$ — W2
$= 1 + 8x + 20x^2 + 16x^3 + 4x^4$ — W1

(iii)

	1	2	3	4	5
A		X			
B	X	X	X		
C				X	
D				X	X
E				X	

M1 W1

Rook polynomial for excluded positions $= 1 + 8x + 20x^2 + 16x^3 + 4x^4$ — MW1
Rook IE theorem — M1
Number of positions $5! - 8 \times 4! + 20 \times 3! - 16 \times 2! + 4$ — M1 W1
= 20 positions — W1 — 14

		AVAILABLE MARKS
5 **(a)** **(i)** $P_G = \frac{1}{14}(x_1^7 + 6x_7^1 + 7x_1^1 x_2^3)$	M1 W1	
(ii) Replace x_i by 3	M1	
Number of necklaces		
$= \frac{1}{14}(3^7 + 6 \times 3 + 7 \times 3^4)$	MW1	
= 198 necklaces	W1	
(b) **(i)** $P_G = \frac{1}{24}(x_1^8 + 6x_4^2 + 9x_2^4 + 8x_1^2 x_3^2)$		
Replace x_i by $R^i + B^i$	M1	
$P_G = \frac{1}{24}\{(R+B)^8 + 6(R^4+B^4)^2 + 9(R^2+B^2)^4 + 8(R+B)^2(R^3+B^3)^2\}$	M1 W1	
(ii) Terms in R^5B^3:	M1	
$\frac{1}{24}({}^8C_5R^5B^3 + 8 \times R^2 \times 2R^3B^3)$	MW3	
$= \frac{1}{24}\left(\frac{8 \times 7 \times 6}{6} + 8 \times 2\right) R^5B^3$		
$= 3R^5B^3$		
so 3 colourings	W1	13

6 (i) Introducing slack variables M1

$2x + y + u = 10$

$x + 3y + v = 12$

$-2x - 3y + P = 0$ MW2

AVAILABLE MARKS

(ii)

2	1	1	0	0	10
1	3	0	1	0	12
–2	–3	0	0	1	0

MW3

(iii) First iteration – Identify pivot column (2) and row (2)
(as 12/3 < 10/1) M1 MW1
Divide pivot row by 3:

2	1	1	0	0	10
1/3	1	0	1/3	0	4
–2	–3	0	0	1	0

MW1

Reduce other row entries in pivot column to zero by subtracting
k × pivot row

5/3	0	1	–1/3	0	6
1/3	1	0	1/3	0	4
–1	0	0	1	1	12

MW1

Second iteration – identify pivot column (1) row (1) $(6 \div \frac{5}{3} < 4 \div \frac{1}{3})$ MW1
Divide pivot row by 5/3

1	0	3/5	–1/5	0	18/5
1/3	1	0	1/3	0	4
–1	0	0	1	1	12

MW1

Reduce other row entries in pivot column to zero by subtracting $k \times$ pivot row

AVAILABLE MARKS

1	0	3/5	–1/5	0	18/5
0	1	–1/5	2/5	0	14/5
0	0	3/5	4/5	1	78/5

MW1

(iv) Optimal solution from above tableau:
$x = 3.6$, $y = 2.8$, $P = 15.6$ M1 W2 16

Section D **75**